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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

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THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XIII, Number 1

Edited by

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This bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

BROWN UNIVERSITY MATHEMATICAL PRIZES FOR FRESHMEN

MARY E. DECHERD
University of Texas

Out of gratitude and respect to his Alma Mater, an alumnus of Brown University has established a fund known as the Brown University Mathematical Prize Fund, from the interest of which prizes are awarded annually by the staff of the Department of Pure Mathematics on the basis of competitive examinations.—*University of Texas Catalogue.*

The donor provides for four prizes, one of which is offered to students of the calculus, while the other three are given for excellence in high-school mathematics.

These prizes are offered to the regular freshmen making the best grades on a special voluntary examination to be held on the afternoon of the second Saturday in October. The examination will cover the minimum entrance requirements in mathematics, namely, elementary algebra and plane geometry.—*University of Texas Catalogue.*

This year the examination for freshmen was held on October 13. That considerable interest was felt in the contest is evidenced by the fact that more than thirty students undertook the examination.

The questions follow:

1. Solve: $(2^x)^x = \dfrac{1}{2^{-7x+12}}$.

2. At what time are the hands of a watch at right angles between 2 and 3 o'clock?

3. Two equal circles intersect so that the common chord equals the radii of the circles. Find the area of the figure that lies within both circles.

4. Draw a circle tangent to two given parallel lines and passing through a given point.

First Prize: Helen Hill, Mexia High School.

Second Prize: F. Burton Jones, Albany High School.

Third Prize: Marguerite Oberkampf, Jefferson Davis High School, Houston.

Number of students solving Question 1	3
Number of students solving Question 2	0
Number of students solving Question 3	2
Number of students solving Question 4	3

A PRESCRIBED COURSE FOR FRESHMEN AND THEIR MATHEMATICAL HABITS

H. E. Bray
Rice Institute, Houston, Texas

From all reports it appears that college teachers all over the country are worried about the defects in preparatory training of the boys and girls who come to college. This is an old story. We, at Rice Institute, are doing our share of worrying. We all know, or think we know, the reasons for these defects. The main reason perhaps is epitomized in the current expression, "mass production." The elementary and high schools are evidently forced to produce huge numbers of graduates, and large numbers of these are swarming to the colleges—in fact they have nowhere else to go. There are not enough good school teachers available to obtain very good results with such numbers of students. Consequently the average high-school graduate is bound to be a kind of "model" product, if you will excuse the metaphor. He has lately had a sort of prosperous look, graceful lines, Duco finish, full balloons, and other refinements, but he cannot go up Pike's Peak "in high." It will take a long time to produce a new and better model. We must take the freshman as we find him, for there is no prospect of getting anything better at present out of the high schools.

In order to get better results with our freshmen at Rice we have lately instituted a new scheme—new to Rice—for teaching the prescribed course in first-year mathematics. The plan is now in its second year.

The course is taught in six sections of from 70 to 100 students each. Each section has three classes per week which are two hours long. The method of instruction is a sort of "supervised study," the purpose being to let the student learn by doing mathematics rather than by watching someone else do it. The student performs his exercises under the supervision and with the assistance of a corps of assistants. We have one assistant or instructor to every

twenty or twenty-five students. The class begins with an explanatory lecture by the instructor in charge; this lasts about half an hour. The rest of the time the students sit at drafting tables working exercises and problems. When they need help they ask for it of the assistants who circulate continually among them giving helpful advice and criticism on the manner in which the work is being done. I may add that, of all things, a good textbook is an essential of this method of instruction, for the reason that it is intended that no small part of the instruction is obtained by the student himself by studying the book.

Here are some of the advantages, as we see it, of the method over our former lecture and recitation method:

1. Every student enrolled in the course has to work.
2. Bad habits of work are more likely to be discovered and corrected early in the course.
3. Immediately after his explanatory lecture the instructor is able to judge how well the subject has "got across"; he can then give supplementary explanations just when they are most needed.
4. The instructor soon gets acquainted with his students personally, and vice versa; there is little formality. The instruction is therefore personal, *i.e.* help is given in the form that fits the needs of each particular student.
5. Many students feel an added incentive to work, which is due perhaps to the air of industry which is noticeable in the classroom. The loafer either gets to work or drops the course.
6. The brighter students need less help than others, especially help of the more elementary sort. They can usually cover more ground, solve harder problems, by themselves without having to listen to tiresome explanations suitable for students of lower grade. Mediocre students have a chance to get a new grip on the subject, through personal instruction, and sometimes show gratifying improvement. The hopeless students can be weeded out early and put into a preparatory class for further training so that they can take the prescribed course the next year with some hope of success.

The subject matter of the course proper comprises trigonometry, analytic geometry, and calculus, and it was originally intended to devote one-third of the year to each subject. But by the time we arrive at the theory and applications of logarithms in the trigonometry we have been able to convince the students that their algebra needs to be renovated. We therefore insert at this point about ten lessons in elementary algebra, designed to correct their bad habits of work, and their lack of understanding of fundamental principles, which have been noted at that time. (This insertion, of course, unfortunately shortens by ten lessons the amount of time available for calculus at the end of the year.) The method of instruction makes it very easy to determine the topics in elementary algebra which need to be emphasized.

Here are some of the bad habits of work most frequently noticed:

1. Studying the textbook without using a pencil; trying to learn mathematics without doing it.
2. Working exercises with no attention to principles; merely getting the answer by mechanically applying rules of thumb.
3. Trying to do too much of the work in the head, when solving problems, instead of writing down the successive steps of the argument which are involved in arriving at a solution. (I believe that many school teachers set a high premium on a student's ability to take two or three steps in one. This often has the effect of getting the student into a brown study in which he wastes much time and finally writes down something entirely wrong.)

Here are some of the stumbling blocks frequently noted which seem to indicate a lack of understanding of principles:

1. The meaning of parentheses:
   (a) $6a-5(a+2)=6a-5a+10$.
   (b) Subtract $5x-7$ from $8x$; $8x-5x-7$.

2. Fractions: (Annihilation of the denominator). Confusion between

(a) Simplify: $\frac{x-1}{x+2} + \frac{3x-1}{x-3}$

and

(b) Solve: $\frac{x-1}{x+2} + \frac{3x-1}{x-3} = 0.$

3. Fractions: (Complex): $\frac{\frac{1}{2} + 3x}{7 + \frac{4x}{5}}.$

4. Fractions: (Improper cancellations): $\frac{8+9x}{8+11x}.$

5. Use of sign = as a symbol of connection (conjunction):

$$2x^2-6x-8 = 0.$$
$$= x^2-3x-4 = 0.$$

$$582.7 \times 3.964 = \log\ 582.7 + \log 3.964.$$

6. Meaning of "solve an equation." Inability to solve for anything more complicated than a single letter:

Solve for m/n: $am+bn+cm+dn = 0.$

7. $(x+2)(x-3) = 0$; then what?

8. Factoring of quadratic trinomials. *(a)* By guessing rational factors:

x^2-2x-3 can be factored;
x^2-2x-4 cannot.

The method of completing the square has so many uses in all branches of mathematics that every opportunity should be grasped to make the pupil familiar with it, *e.g.*, in factoring quadratics.

9. A fourth of our freshmen cannot solve a quadratic equation by any method.
10. Exponents. Meaning of $\sqrt{5}$, $x^{\sqrt{5}}$, $x^{\sqrt{5}} \cdot x^{\sqrt{5}} = ?$

EXAMINATIONS IN HIGH-SCHOOL ALGEBRA AND GEOMETRY GIVEN TO FRESHMAN MATHEMATICS CLASSES

MARY E. DECHERD
University of Texas

A number of times in previous years, examinations in high-school algebra and geometry have been given to our freshman students. Sometimes the Department of Pure Mathematics as a whole has required these examinations, but more frequently, it has been left with the individual instructor as to whether he would examine his sections.

These examinations have been given with the two-fold object of revealing to the instructor the needs of his students and of enabling the student himself to realize and overcome his deficiencies.

This fall by vote of the Department of Pure Mathematics, it was decided that each freshman section should be given an examination in high-school work. Instructions were also given each student to record on his paper the name of the high school from which he received his last training in mathematics. Each instructor was responsible for the grading of the papers from his sections. These grades were then turned over to a member of the department, and were filed and classified according to high schools.

Perhaps a remark concerning the character of the examinations would be of interest. While the examinations given to the various sections were not uniform as is the case in some of the departments in the University, an effort was made to secure comparatively uniform difficulty in examinations. A typical examination was written on the board, and each instructor suggested changes that should be made in the questions. More questions than could be given in any one examination were thus brought before the entire group. The examinations actually given were expected to be modelled on the questions discussed in the group.

Of course, conclusions drawn from the results of these examinations will be of slight value; but if the same plan is continued for several years, as in the case of the Department of English, statistics of considerable value will be afforded.

The following points are of interest:

Number of schools from which students come	355
Number of students taking the examination	880
Number of students failing on the examination	571
Number of students passing the examination	309
Per cent of students passing the examination	33.9

A few more general remarks may be made concerning the the results of the tabulation. There are only eighty-two high schools with three or more taking the examination. One high school shows seven students failing out of the seven who took the examination, while there are three schools in which six failed out of six taking the examination. On the other hand among the schools which had five students taking the test is Pearsall with four out of five passing. This record of 80 per cent is the best made by any school with five or more students.

Among the schools with larger numbers of students, Galveston leads with 76.9 per cent of its students passing the examination, while Austin ranks second with 53.7 per cent.

While the students have been informed of the grades which they have made on these tests, there is no plan to form so-called "Zero Sections" of those who have failed to make 60 per cent. It is hoped that by consistent review of their elementary algebra and geometry, these students will be able to overcome their deficiencies.

Some of the questions given are appended.

Simplify: (1) $(a^{2x} \cdot a^{c})a^{xt}$. (2) $\sqrt[3]{81}$.

(3) $\dfrac{1}{\sqrt{3}}$. (4) $\dfrac{1}{\sqrt{3}-\sqrt{2}}$.

(5) $\left(\frac{1}{x}-\frac{1}{y}\right) \div \left(\frac{1}{x}+\frac{1}{y}\right)$.

Solve for x and y: $2x-y=1$,
$x+2y=2$.

Solve for x: (1) $\frac{2}{x}-\frac{1}{x}-\frac{3}{x}=7$.

(2) $\frac{1-x}{2-x}=2$.

(3) $(x-1)(x+2)=0$.

(4) $\sqrt{x-1}=2$.

(5) $3x=x^2$.

(6) $3x^2-17x+10=0$.

If twice one number is half another, and the sum of these numbers is 4, find the numbers.

Express 1/11 as a decimal.

Factor completely: x^4-81.

Simplify: (1) $(1+\sqrt{x})(1-\sqrt{x})$.

(2) $(x^{\frac{1}{2}}-y^{-\frac{1}{2}})(x^{\frac{1}{2}}-y^{\frac{1}{2}})$.

(3) $\dfrac{\frac{a}{a-b}-\frac{a}{a+b}}{\frac{b}{a-b}+\frac{b}{a+b}}$.

(4) $\dfrac{h^2-k^2}{h-k}$.

State and prove one theorem concerning the congruence of triangles.

Given a right triangle with its two perpendicular sides 6 and 8, find the perpendicular sides of the similar right triangle whose hypotenuse is 5.

Solve the equation $Ax+D = Bx-A$.

Solve for a the equation $ca - \dfrac{b}{a} = d$.

PHYSICS AND HIGH-SCHOOL MATHEMATICS

L. W. Blau
University of Texas

Some critics of our schools, especially teachers of science, have long contended that our college students are sadly deficient in arithmetic and simple, elementary algebra. It is also maintained by many that the freshmen who have entered college during the past two or three years were not as well prepared as those were who entered seven or eight years ago and previous to that time; in other words, it is claimed that our high schools are turning out an increasingly inferior product.

The fact that most of the freshmen enter college without having to pass entrance examinations, merely on the reputation of the high schools from which they graduated, renders the elimination of poorly prepared students impossible. Furthermore, there is no opportunity for determining the extent of their deficiencies in mathematics without giving special tests. Such examinations have been given to students in freshman physics at the University of Texas, and the results of most of these tests are presented on the following pages.

In February of this year, at the beginning of the spring semester, a test was given to a class of freshmen who were then beginning the study of physics. Seventy-three handed in papers. The results are surprising. The students were given an opportunity to visit special classes in arithmetic and elementary algebra, and after about three weeks were given another test on which they did a little better. An article entitled "Elementary Mathematics and the High-School Graduate" giving the results of these examinations will appear in the December issue of the *Texas Outlook*.

The following test was given to the students in Physics 1 at the beginning of the summer. The results are given for each problem in per cent of the whole class who obtained

correct solutions. There were sixty-three students who handed in papers.

No.	Problem	Per Cent Correct
1.	Add: $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$. Change 5/7 to a decimal.	90
2.	Divide 1.020304 by 0.0004321 to two places of decimals.	63
3.	In the triangle ABC, DE is parallel to AB, and $AB = 6.5$ ft. $AC = 4.5$ ft. $CD = 2.0$ ft. Find DE.	32
4.	Solve for E: $$ER+A = E - \frac{B}{C}.$$	48
5.	Solve for R and S: $4R+3S = 7$, $3R-4S = 8$.	33
6.	Solve for M to two places of decimals: $3M^2+4M-5 = 0$.	11
7.	Extract the square root of 34.5 to two places of decimals.	73
8.	Find the square root of $\frac{4.50}{2}$.	49
9.	Solve for g: $T = 2\pi\sqrt{1/g}$.	19
10.	Given: $$\frac{ax+\frac{b}{y}}{ax-\frac{b}{y}} = 2x-3y.$$ Simplify and solve for y, if $a = b/2$.	4

The average grade was 39.7. It is interesting to note that sixteen members of the class were teachers of mathematics in Texas schools. They had taught arithmetic, algebra, or plane geometry for an aggregate of 192 years. The average grade of these teachers was 43.7, which shows that the teachers know more than the students.

This fall Professors Mather, Kuehne, and Boner cooperated with the writer and tests were given, at the beginning of the fall semester, to all freshmen who had then registered for freshman physics. The tests with the results are given below:

PROFESSOR BONER'S SECTION. 166 STUDENTS

No.	Problem	Per Cent Correct
1.	Add: $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = ?$	90
2.	Change to a decimal: 7/9.	86
3.	Divide 2.0134 by 0.00432 to two places of decimals.	55
4.	In the triangle ABD, EC is parallel to AB, and $AB = 6.5$ ft. $AD = 4.0$ ft. $ED = 1.5$ ft. Find EC.	57
5.	Solve for E: $ER + A = E - \frac{B}{C}$.	30
6.	Solve for R and S: $4R+6S = 21$, $10R-4S = 5$.	48
7.	Solve for M: $3M^2-4M+1 = 0$.	40
8.	Extract the square root of 30.5 to two places of decimals.	66
9.	Solve for g: $T = 2\pi\sqrt{1/g}$.	28

10. Given: $4ax+3by=2y^2$.
Find the value of x, if $a=3$, $b=4$, and $y=8$. 69

The average grade was 56.6; 42 per cent of the students made a grade of 50 or less.

SECTION 2. 118 STUDENTS

No.	Problem	Per Cent Correct
1.	Divide 1.0052 by 0.0034 to two places of decimals.	66
2.	Extract the square root of 22.3 to two places of decimals.	78
3.	Change to a decimal: 2/13.	69
4.	Solve for T_1: $\dfrac{T_1-T_2}{T_1}=E$.	17
5.	Solve for x: $C=a\sqrt{b/x}$.	20
6.	Solve for S: $S^2+4S-2=0$.	11
7.	Solve for M and N: $3M-2N=10$, $2M+N=100$.	47
8.	Add: $1/4+2/5+3/7=?$	73
9.	$\dfrac{\frac{c}{x}+\frac{y}{b}}{\frac{c}{x}-\frac{y}{b}}=14$. Solve for x, if $c=4$, $y=3$, and $b=5$.	28
10.	In the triangle ABC, DE is parallel to BC, and $AB=10$ ft., $DB=4$ ft., $DE=3$ ft. Find BC.	25

The average grade was 45; 67 per cent of the students made a grade of 50 or less.

DR. MATHER'S SECTION. 174 STUDENTS

No.	Problem	Per Cent Correct
1.	Divide 1.23 by 0.000321 to two places of decimals.	64
2.	Add: $2/5+1/2+3/7 = ?$	90
3.	Change to a decimal: $7/17$.	90
4.	Solve for B: $AB+C = \frac{B-A}{C}$.	15
5.	Solve for Q: $5Q^2+3Q-6 = 0$.	6.9
6.	Multiply $2-2/3$ by 1.23.	85
7.	Solve for p: $Z = 4m\sqrt{q/p}$.	21
8.	$\frac{cm-\frac{ar}{2}}{cm+\frac{ar}{2}} = m$. Solve for r, if $c=2$, $m=4$, and $a=3$.	25
9.	Solve for a and b: $5a+7b = 31$, $8a-5b = 1$.	44
10.	In the triangle ABC, ED is parallel to BC, and $AB = 6$ ft. $ED = 2$ ft. $BC = 5$ ft. Find AE.	49

The average grade was 48.8; 65.5 per cent of the students made a grade of 50 or less.

DR. KUEHNE'S SECTION. 93 STUDENTS

No.	Problem	Per Cent Correct
1.	Change to a decimal: 5/11.	94
2.	Divide 0.01234 by 0.0004321 to two places of decimals.	58
3.	Solve for T: $\frac{SR}{M} = \frac{A}{T} + T$.	6
4.	Solve for m: $T = 2\pi\sqrt{k/m}$.	24
5.	Find the length of side a if a, b, c are the sides of a right triangle; c, the hypotenuse $= 25$; $b = 15$.	78
6.	Solve for R: $R^2 - 8R + 7 = 0$.	46
7.	Given: $\frac{ax - \frac{b}{y}}{ax + \frac{b}{y}} = y^2$. Solve for x, if $a=3$, $b=2$, $y=4$.	25
8.	In the triangle ABC, ED is parallel to AB, and $ED = 4$ ft. $AE = 1$ ft. $AC = 2$ ft. Find AB.	34
9.	Add: $2/3 + 1/2 + 4/7 = ?$	78
10.	Solve for A and B: $2A - 7B = 2$, $6A + 4B = 31$.	49

The average grade was 49.8; 64.5 per cent of the students made a grade of 50 or less.

As shown above, 687 students have been tested thus far; that is, we have received papers from that number. There

have been some in each class who did not even hand in a paper; they realized, perhaps, that they did not have much to contribute. There were some, on the other hand, who made zero on the paper submitted. The time allowed for each quiz was one hour. The papers were graded leniently; for instance, in the division by a decimal, a problem was graded correct, if the decimal point was in the right place in the quotient, even though the figures were not all correct. The same is true in changing a common fraction to a decimal. If a student found one root of a quadratic, he was given half credit.

We admit that the student should not be criticised severely for having forgotten how to solve quadratics, because he probably studied algebra in the ninth grade. However, we believe that simple operations like division by a decimal, changing a common fraction to a decimal, the addition of fractions should not be forgotten, because they are used by most people in everyday life. We have found, in our talks with students, that they do not know *why* certain operations are done in a certain manner; they say they have never been told, and they have forgotten the rules which were given them.

The answers which many of these students gave to arithmetical problems indicate very clearly that they have not formed the habit of looking at the answer to see whether or not it seems reasonable. In other words, there is little, if any, attempt to check an answer even very superficially. Too many students divide three by thirteen and obtain twenty-three.

It is hoped that the results of the tests mentioned will arouse our high-school and grammar-school teachers of mathematics to action. This is an alarming condition, and steps should be taken immediately tending toward its improvement. The tests themselves show that it will not do to ignore these results with the remark: "We are not training our students for the university and hence do not intend to develop mathematicians." The man in the street needs more arithmetic than these students know. Why not try these or similar tests on your students?

MAKING USE OF THE CONCEPT OF SYMMETRY IN THE TEACHING OF PLANE GEOMETRY

H. J. Ettlinger
University of Texas

The teacher who "lives" her subject just as the preacher who lives his religion, has no difficulties or problems connected with the practice of her profession. I know a minister who for forty long years has lived in one community and has preached the same kind of life from his pulpit. Though he be neither outstanding orator nor scholar, his sermons command widespread attention.

In teaching plane geometry, by tradition as abstract as the ethic of any theological code, the teacher can make use of everyday life or experience to stimulate the interest of the pupil. One may "live" one dimensional geometry by comparing two lengths by means of a piece of string, by measuring with a ruler, by stepping off or "pacing" a distance on the ground.

If, as it is alleged, the love of symmetry is innate in man, then truly the youthful student may be initiated into some of the mysteries of geometrical truth by capitalizing his interest in an attractive oil cloth pattern or a handsome wall paper design. In fact, symmetry in a line may be readily associated with paper folding. For example, given a straight line and a point not on the line, the symmetrical point in this line may be obtained by folding the paper on the line and pricking through with a pin. By folding along the line joining the given point and its symmetric point, we obtain an "intuitive" method for dropping a perpendicular from a given point to a given line. This exercise forms a splendid introduction to the theorem that the locus of all points equidistant from the extremities of a given line segment is the perpendicular bisector of the given line segment.

If l is the given line and P the given point, one may proceed then to find P', the image of P in l. Let O be the intersection of l and PP', then mark on l, $OQ = OP$ by means of a

circle with O as center and OP as radius. Then we may find the bisector of the angle POQ by folding along a line through O so as to make P and Q images of each other. By continuing this process, we may construct a regular octagon, or a regular polygon of 2^n sides, where n is a natural number. This latter construction joins together with the proposition that the locus of points equidistant from the sides of an angle is the bisector of the angle.

One may, in similar fashion, construct a hexagon from an equilateral triangle, a dodecagon from a hexagon, etc. One may now introduce the notion of *turn* symmetry. For example, if the figure $OPQP'Q'$ be turned so that OP takes the position OQ, then the figure is unchanged or *invariant*. One may illustrate this as follows. Begin with OP in the vertical position and revolve the paper through $90°$ in the positive or counterclockwise direction, then OP assumes the original position of OQ and the figure is unchanged. Such a property would be called a *turn* symmetry. The vertices of an octagon, for example, could be found by turns through an angle equal to $360°/8$.

Another class of theorems which lend themselves to treatment by means of the concept of symmetry and the concomitant process of paper folding is congruence theorems. One may begin with simple situations such as the diagonal of a square dividing it into two equal triangles. The altitude of an isosceles triangle upon the side not equal to another side divides the given isosceles triangle into two equal triangles. Such theorems might be multiplied in number without end.

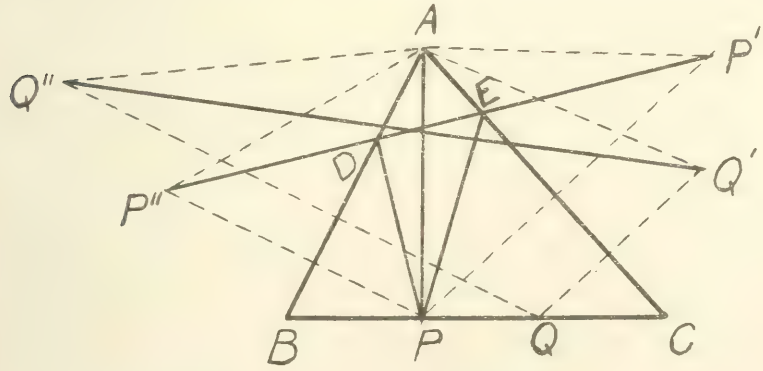
An interesting construction which may be solved by the principle of symmetry or paper folding is the following: Given two points A and B not on a line l but on the same side of l, to draw a straight line from A to a point C on l, thence a straight line from C to B so that $AC+CB$ shall be as short as possible. The solution is the following: Fold on l, and mark A'. Then draw $A'B$ and let C be the intersection of $A'B$ and l, then $AC+CB$ is the required length. The proof is left to the reader.

Not only is the notion of symmetry an entrancing and fascinating one, but as a matter of fact, it ties up with an aspect of mathematical discipline called group theory, which is one of the most fundamental of all mathematical concepts, leading to the most basic ideas of the theory of matter and energy, namely quantum dynamics and atomistics.

I shall close with a problem which illustrates a sequence of several successive applications of the use of symmetry. According to Professor Constantin Caratheodory, of the University of Munich, who visited in Austin last summer, this proof was first given by Fejer, a Hungarian mathematician, while a young student. The problem is much older, but this was the first elementary proof of it.

Problem.—Given an acute-angled triangle, to inscribe a second triangle in the first such that the perimeter is as small as possible.

A triangle $A'B'C'$ is inscribed in another triangle ABC if each vertex of the first triangle is on a side of the second and no side of the second triangle contains two vertices of the first. *The solution is the triangle whose vertices are the feet of the altitudes.*



Proof.—Let ABC be the given triangle.

Let P' and P'' be the images of P in AC and AB respectively. Then perimeter of $DEP = P''D + DE + EP'$. For P fixed, the solution is such that $P'P''DE$ is a straight

line. Now consider any other point Q other than the foot of the altitude, and its images Q' and Q'', then

angle $Q'AQ'' =$ angle $P'AP''$

and $AP < AQ$.

But $AQ' = AQ'' = AQ$, hence $P'P'' < Q'Q''$.

THE CURRICULUM IN ALGEBRA

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The teacher of first-year algebra in the Texas high schools faces a new situation this year: Wentworth's *New School Algebra*, which has been used as the textbook for longer than most of us can remember, now yields its place to one of four new elementary texts (Wells and Hart; Hawkes, Luby, and Touton; Edgerton and Carpenter; and Thorndike). During recent years there has been much discussion in educational circles of the curriculum in algebra, and while some questions are still in dispute, general agreement has been reached on many points, such as the desirability of cutting down the amount of formal manipulation, omitting short cuts and special methods that have little application, and emphasizing the topics that require independent thinking or have genuine applications. Algebra cannot be made as completely a course in reasoning as geometry can (and should), for a certain amount of drill on formal processes is essential, but a comparison of any one of the new texts with the old one will show that a long step has been taken in this direction. Care has also been devoted in the newer books to the better correlation of algebra with arithmetic and with the life and interests of the pupil.

Nearly all textbooks include more material than any individual teacher is likely to use. From the standpoint of the author and the publisher, who want large sales, it is naturally desirable to include everything which may possibly be needed in any school, though some of them are considerate enough to indicate which parts can best be omitted. A selection of material is therefore necessary, and this article is written in the hope of aiding teachers of elementary algebra in Texas to make this selection and to distribute suitably the time available among the different topics.

The topics to be studied and the number of lessons which should be devoted to each were among the subjects discussed in a course on "Teaching Problems in Mathematics" given by the author in the last Summer Session of the University of Texas, which was attended by a number of experienced high-school teachers. The result of the discussion was a general agreement on the following tentative outline for the first year's work in algebra.

Topic	Lessons
Literal numbers, formulas, and negative numbers	25
Addition and subtraction	5
Parentheses	3
Multiplication	10
Division	6
Simple equations	8
Products and factoring	20
Fractions and fractional equations	20
Graphs	10
Simultaneous linear equations	10
Radicals	8
Quadratic equations	12
Total	137

The total of 137 lessons gives some leeway for reviews and extra work which certain classes may need on some topics. It is of course not desirable to force all classes to follow a rigid schedule, but the teacher is almost certain to obtain better results if he maps out his work in advance and knows approximately how much time should be devoted to each subject. His schedule can be modified from year to year in the light of his experience, and to fit the needs of classes of different ability.

A similar discussion of the second year's work in algebra led to the following outline.

Topic	Lessons
Fundamental operations	10
Simple equations	5
Factoring	15
Fractions and fractional equations	20
Functions and graphs Simultaneous linear equations	15
Exponents	10
Radicals and radical equations	30
Quadratic equations	15
Theory of quadratic equations	5
Simultaneous quadratics	10
Binomial theorem	5
Ratio, proportion, and variation	10
Total	150

It seems desirable to add a few lessons on trigonometry at the end of the second year's work if time permits, because of the large number of applications which it has to practical problems, such as heights and distances, which are easily comprehended by the pupil. It is not necessary to go very far into the subject, for as soon as the student has learned how to solve right triangles a rich field of applications is open to him. Most recent textbooks for the second year's work include a chapter on trigonometry.

TO LOCATE THE GOLDEN SECTION POINT OF A GIVEN SEGMENT

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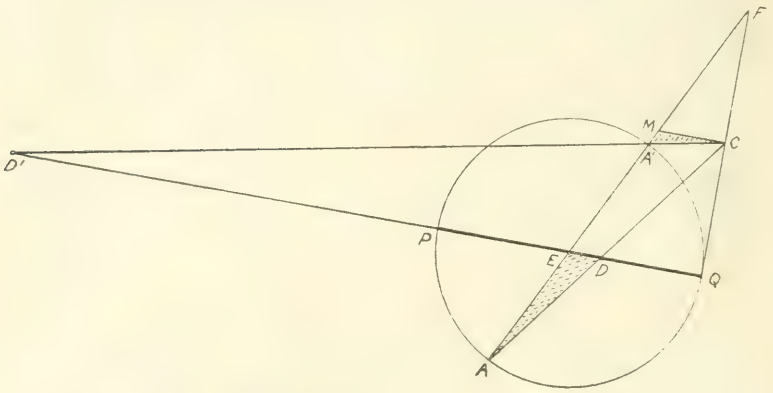

Given: The segment PQ.

Required: 1. A point D such that $PD = \sqrt{5}-1$.
2. A point D' such that $PD' = \sqrt{5}+1$.

Construction: On PQ as a diameter describe a circle with center E. Erect QF perpendicular to PQ and equal to PQ. Draw FE cutting the circle in the points A' and A. From C, the midpoint of QF, draw CA intersecting the segment PQ in D. Then D is the internal point of division.

Draw CA' and extend it to meet PQ prolonged in the point D', then D' is the external point of division.

Proof that D is the internal point:

Draw CM parallel to PQ.

From the similar triangles AED and AMC we get

$$\begin{aligned} AE : ED &= AM : MC \\ &= 2AM : 2MC \\ &= 2(AE+EM) : EQ \\ AE : ED &= AA'+EF : EQ \end{aligned}$$

Now $EF = \sqrt{5}$ and $AA' = 2r : EQ = AE = r = 1$

$$\begin{aligned} ME-ED : EA &= 3AE-EF : 2EQ-EF \\ PD : r &= 3+\sqrt{5} : 2-\sqrt{5} \end{aligned}$$

$$\therefore PD = \frac{3-\sqrt{5}}{2+\sqrt{5}} = \sqrt{5}-1.$$

Q. E. F.

Proof that D' is the external point:
From the similar triangles DEA' and MCA' we get

$$\begin{aligned} D'E : EA' &= MC : A'M \\ &= 2MC : 2A'M \\ &= EQ : 2(EM - A'E) \\ &= EQ : EF - 2A'E \\ \therefore D'E : r &= r : \sqrt{5} - 2r \\ \therefore D'E + r : r &= \sqrt{5} - r : \sqrt{5} - 2r \\ \therefore D'Q &= \frac{\sqrt{5} - 1}{\sqrt{5} - 2} = 3 + \sqrt{5} \\ PD' &= 1 + \sqrt{5}. \end{aligned}$$

Q. E. F.